A model of CP Violation from Extra Dimension

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We construct a realistic model of CP violation in which CP is broken in the process of dimensional reduction and orbifold compactification from a five dimensional theories with $SU(3)\times SU(3)\times SU(3)$ gauge symmetry. CP violation is a result of the Hosotani type gauge configuration in the higher dimension.

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Introduction

There are many mechanisms of obtaining CP violation if one starts from a CP conserving higher dimensional theory[1, 2]. The idea is not new. For example, Thirring considered such possibility as early as 1972[1]. Recently, with the renewed interest in the extra dimensional theories due to many new approaches to the additional dimensions, new schemes of obtaining CP violation from a CP conserving higher dimensional theory were proposed. More recently**, for example, in Ref.[3], CP violation arises as a result of compactification due to the incompatibility between the orbifold projection condition that defines the projected geometry of the space and the higher dimensional CP symmetry. Therefore, the origin of CP violation can be geometrical in nature.

Another interesting geometrical scheme has been pursued in Ref.[4]. In this scheme, CP violation arises out of the possibility that the high dimensional gauge field may develop a nontrivial configuration when compactified on an orbifold type of geometry with nontrivial topological loops. Using such configuration to break gauge symmetry is called the Hosotani mechanism [6]. It was initially proposed to break gauge symmetry, however, it was realized [4] that it can also be used to break CP symmetry. In Ref.[5], several interesting models were pursued in this direction. They include one with $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ gauge symmetry and another** one with $SU(4)\times U(1)$ gauge symmetry in the higher dimensional theory. Some prospects of grand unification in the high dimension were also discussed. The models aim to produce the Kobayashi-Maskawa Model in four dimension.

One weakness in the models proposed in Ref.[4, 5] is that the Hosotani vacuum expectation value are used to break the electroweak $SU(2)_L$ symmetry. Since the Hosotani vacuum expectation value is related to the ground state value of the Wilson loop integral over the compactified dimension, as we shall illustrate in the next section, it is expected that its value should be of the size of inverse of the extra dimension, R^{-1}. Since the current experimental limit on R^{-1} is larger than the weak scale already, it is preferable to have a model in which the Hosotani mechanism while breaks CP and gauge symmetry but does not involve in $SU(2)_L$ electroweak breaking.

In this paper, we propose a higher dimensional model in which the Hosotani mechanism breaks CP and higher dimensional gauge symmetry. The electroweak gauge symmetry is broken in four dimension by a zero mode which is $SU(2)_L$ scalar doublet. To achieve this goal, we employ the trinification gauge group of $SU(3)^3$ in higher dimension in which the extra fermions are naturally needed. In particular, in the quark sector, the new fermions are the extra vectorial down type quarks, $D_{L,R}$. The Hosotani mechanism induces CP violating mixing between the light quark and the heavy down quark as well as breaking the $SU(3)^3$ gauge symmetry. Such mixing results in four dimensional Kobayashi-Maskawa CP violation among the light quarks.

At the bottom of the issue, the CP violation arises because the Hosotani vacuum expectation, being related to the Wilson line of the gauge configuration, is pseudoscalar and effectively CP-odd in nature in four dimension. In some simple cases, such CP violating phase can be rotated away, but in general it can not.

HOSOTANI BREAKING AND ITS SCALE

If the space is not simply connected, the gauge field can develop a vacuum expectation value along a noncontractable loop in the extra dimension and it cannot be gauged away[6]. The VEV of this gauge field can give rise to a mass term for fermions (called “Hosotani mass term”):

$$-ig\bar{\psi}\langle A_y\rangle\psi. \tag{1}$$

It is not hard to show that the magnitude of the Hosotani mass term should** be, in general, of the order of the compactification scale. For example, consider** a $SU(N)$ gauge theory on a five dimensional non-simply connected space $M^4\times S^1$, where M^4 is Minkowski space and the radius of circle S^1 is R, with N_f flavor fermions in fundamental representation. The** 1-loop effective

potential takes the form [6, 7]:

$$V_{\rm eff}(\langle A_y\rangle) = \frac{3}{128\pi^7 R^5}\left\{-3\sum_{i,j=1}^{N} F_5(\theta_i-\theta_j) + 2^2 N_f \sum_{i=1}^{N} F_5(\theta_i)\right\}, \quad (2)$$

where $F_5(x) = \sum_{n=1}^{\infty}\cos(nx)/n^5$ and gauge vacuum expectation values are parameterized as $\langle A_y\rangle = (2\pi g_5 R)^{-1}{\rm diag}(\theta_1,\theta_2,\cdots,\theta_N)$. In the r.h.s of (2), the first and second terms are the gauge-ghost and fermion 1-loop contributions**, respectively. Since the $F_5(x)$ is a cosine-like function with period 2π, the nonzero** minimum of (2) tends** to have a minimum at $\theta_i \sim \mathcal{O}(1)$ (or $\langle A_y\rangle \sim R^{-1}$) unless there are extra fine-tunings.

Therefore, when we consider models with compactification radius much smaller than electroweak scale, it is unnatural to use Hosotani mechanism to break $SU(2)_L$ gauge symmetry.

THE $SU(3)_c\times SU(3)_l\times SU(3)_r$ MODEL

In this paper we propose a $SU(3)_c\times SU(3)_l\times SU(3)_r$ gauge theory[8] which is assumed to be CP symmetric in $4+1$ dimension. Orbifold symmetry breaking mechanism breaks the gauge symmetry to $SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R$ when the space is compactified on orbifold to a 3+1 dimension. The zero modes of the compactification, serving as the four dimensional scalar bosons, further break the symmetry to Standard Model group and then to $U(1)_{em}$.

Let's first list the basic field contents of this model in $4+1$ dimension:

	$SU(3)_c\times SU(3)_l\times SU(3)_r$
$A_{c,M}$	$(8,1,1)$
$A_{l,M}$	$(1,8,1)$
$A_{r,M}$	$(1,1,8)$
Q_l	$(3,3,1)$
$\bar{Q}_r$	$(\bar{3},1,\bar{3})$
L	$(1,\bar{3},3)$
Φ_l	$(1,8,1)$
Φ_r	$(1,1,8)$
$\Sigma_{\bar{l}r}$	$(1,\bar{3},3)$

where the index $M=(\mu,y)$ runs from 0 to 4, μ from 0 to 3 and y is the fourth dimension. Note that an irreducible fermion in $4+1$ dimension contains fermions of both chirality in $3+1$ dimension. In this trinification model, the gauge fields A_c,A_l, and A_r are in the adjoint representation of their perspective $SU(3)$. The fermionic fields $Q_l(\bar{Q}_r)$ contain the standard model left-handed (right-handed) quarks and their chiral partners as required in a $4+1$ dimensional theory. The lepton multiplet, L, contains the leptonic sector of the standard model and additional leptons (to be discussed later) as well as their chiral partners. The scalar fields Φ_l, Φ_r and $\Sigma_{\bar{l}r}$ are needed to give masses to particles. One also notes that this model can naturally be embedded into a grand unified group, E_6, if so desired.

To break gauge symmetry by geometry through the Hosotani mechanism, we compactify the $4+1$ dimensional space on orbifold. Orbifold is produced by imposing projection condition on the space and the fields. This projective symmetry dictates a transformation on each field and selects the zero modes which will serve as the low energy modes that play the active role in $3+1$ dimensional theory. In this paper, we consider the simplest case in which there is only one extra dimension with a Z_2 projection, i.e., S^1/Z_2. It is the circle with the points identification under the parity operation in the fourth dimension ($y\to -y$).

Now we have to specify the Z_2 representation of each field. Note that, for any transformation under Z_2, we are allowed to insert the transformation matrix which belongs to symmetry of the theory, such as a discrete gauge transformation $P_{\mathcal{G}}\in\mathcal{G}$ (with $P_{\mathcal{G}}^2=\mathbf{I}$).

So we have boundary conditions

$$\begin{aligned} A^a_\mu(x^\nu,y)\,\lambda^a &= A^a_\mu(x^\nu,-y)\,P_{\mathcal{G}}\lambda^a P_{\mathcal{G}}^{-1}, \\ A^a_y(x^\nu,y)\,\lambda^a &= -A^a_y(x^\nu,-y)\,P_{\mathcal{G}}\lambda^a P_{\mathcal{G}}^{-1}, \end{aligned}$$

for gauge fields and

$$\Psi(x^\mu,y) = P_{\mathcal{G}}\gamma_5\Psi(x^\mu,-y),$$

for fermions [9].

The transformation properties for scalars are determined by their couplings to fermions. Since, under the Z_2 transformation, $\bar{\Psi}_i\Psi_j$ term transforms into $-\bar{\Psi}_i\Psi_j$, the scalar fields must transform as

$$\Phi^a(x^\mu,y)\,\lambda^a = -\Phi^a(x^\mu,-y)\,P_{\mathcal{G}}\lambda^a P_{\mathcal{G}}^{-1},$$

for the scalar boson to couple to the fermion. An adjoint scalar Φ, which couples to fermions, and the fourth component of the gauge field A_y must gets the same zero modes after compactification. To illustrate the orbifold projection, let's first consider the case of only one $SU(3)$. The representation decomposes as follows:

$SU(3)$	$\supset\ SU(2)\times U(1)$
3	$\to 2_1+1_{-2}$
8	$\to 3_0+1_0+2_3+2_{-3}$

here we have used $P_{\mathcal{G}}={\rm diag}(-1,-1,1)$ as the appropriate projection. We verify easily that $P_{\mathcal{G}}$ commutes with generators of an $SU(2)\times U(1)$ subgroup, while anticommuting with the other generators, say $[\lambda_{1,2,3,8}\,,P_{\mathcal{G}}]=0$,

$\{\lambda_{4,5,6,7}\,, P_{\mathcal{G}}\} = 0$. As a result, the zero mode gauge fields are:

$$\begin{aligned} A_\mu^{\bar{a},(0)} &\rightarrow 1_0 + 3_0, \quad (\bar{a} = 1,2,3,8) \\ A_y^{\hat{a},(0)} &\rightarrow 2_3 + 2_{-3}, \quad (\hat{a} = 4,5,6,7). \end{aligned}$$

The fermions in the 3 representation reduce to the following zero modes

$$\left(u_L^{(0)}\, d_L^{(0)}\, B_R^{(0)} \right) \leftrightarrow 2_1 \oplus 1_{-2}. \tag{3}$$

Here and from now on, L, R represent the chirality of the $3+1$ dimensional fermions.. Zero modes of adjoint scalar Φ coupled to fermions have the same gauge quantum numbers as A_y.

More precisely, since $(3,3,1) \mapsto (3,2,1) \oplus (3,1,1)$, we can write, for example, Q_l as :

$$Q_l = \begin{pmatrix} u_l^\alpha \\ d_l^\alpha \\ B_l^\alpha \end{pmatrix} \Rightarrow_{orbifolding} \begin{pmatrix} u_{lL}^{\alpha,(0)} \\ d_{lL}^{\alpha,(0)} \end{pmatrix} \oplus B_{lR}^{\alpha,(0)},$$

where $\alpha = 1,2,3$ is the $SU(3)_c$ group index, the superscript (0) denotes the zero modes in $3+1$ dimensions. Similar notations can be used to $\bar{Q}_r$ field. Note that here we use l, r to denote gauge groups while using L, R to denote the handedness of the fermions. ¿From now on, we neglect the color index α and zero mode label (0).

Back to trinification model, in order to break $SU(3)_l \times SU(3)_r$ down to $SU(2)_l \times SU(2)_r \times U(1)_l \times U(1)_r$ by orbifolding, we choose our projection operator $P_{\mathcal{G}}$ as

$$P_{\mathcal{G}} = \mathrm{diag}(-1,-1,1)_l \otimes \mathrm{diag}(1,1,-1)_r. \tag{4}$$

After the projection, the field content of the zero modes of the theory in $3+1$ dimension becomes:[10]

	$SU(3)_l \times SU(3)_r$	$SU(2)_l \times SU(2)_r$ $\times U(1)_l \times U(1)_r$
$A_{l\mu}$	$(8,1)$	$(3,1)_{(0,0)} + (1,1)_{(0,0)}$
A_{ly}		$(2,1)_{(3,0)} + (2,1)_{(-3,0)}$
$A_{r\mu}$	$(1,8)$	$(1,3)_{(0,0)} + (1,1)_{(0,0)}$
A_{ry}		$(1,2)_{(0,3)} + (1,2)_{(0,-3)}$
$\begin{pmatrix} u_l \\ d_l \end{pmatrix}_L$	$(3,1)$	$(2,1)_{(1,0)}$
B_{lR}		$(1,1)_{(-2,0)}$
$\begin{pmatrix} u_r^c \\ d_r^c \end{pmatrix}_R$	$(1,\bar{3})$	$(1,\bar{2})_{(0,-1)}$
B_{rL}^c		$(1,1)_{(0,2)}$
L_L	$(\bar{3},3)$	$(\bar{2},2)_{(-1,1)} + (1,1)_{(2,-2)}$
L_R		$(\bar{2},1)_{(-1,-2)} + (1,2)_{(2,1)}$
ϕ_l	$(8,1)$	$(2,1)_{(3,0)} + (2,1)_{(-3,0)}$
ϕ_r	$(1,8)$	$(1,2)_{(0,3)} + (1,2)_{(0,-3)}$
$\sigma_{\bar{l}r}$	$(\bar{3},3)$	$(\bar{2},2)_{(-1,1)}$
χ		$(1,1)_{(2,-2)}$

where the $SU(3)_l \times SU(3)_r$ origins of the fields are also included in the middle column. Note that B_{lR} and B_{rL}^c form a vector-like quark pair singlet under $SU(2)_L$. We can identify B_{lR} and B_{rL}^c as heavy quarks $D_{L,R}$.

Because the scalar fields Φ_l, Φ_r and $\Sigma_{\bar{l}r}$ transform differently under Z_2, their respective zero mode fields in $3+1$ dimension after the orbifolding also have different transformation property.

CP VIOLATION

In order to make a mass term for quarks, some scalar field must develop VEVs and break $SU(2)_l \times SU(2)_r$. In this model, we assume that ϕ_r and χ develop VEVs. $\langle\phi_r\rangle$ and $\langle\chi\rangle$ break the $SU(3)_c \times SU(2)^2 \times U(1)^2$ to $SU(3)_c \times SU(2)_L \times U(1)_Y$. In order to get M_D (mass term for $\bar{B}_{lR} B_{rL}^c$), we need a standard model singlet field χ to survive after orbifolding. That's why we need $\Sigma_{\bar{l}r}$ field. Note that the VEV of χ breaks $U(1)_l \times U(1)_r$ to $U(1)_Y$ and gives $M_D \sim f_\Sigma \langle\chi\rangle$. In order to couple $\bar{d}_r$ with B_{rL}, we need the $SU(3)_r$ scalar field ϕ_r to develop VEV along the λ_6 direction, say $\langle\phi_r\rangle = \langle\phi_{r6}\rangle = v_r \lambda_6$. $\langle\phi_r\rangle$ is related to the $SU(2)_r$ breaking scale. In that case, the Hosotani mass term $\langle A_{ry}\rangle$ which is assumed to develop a nonzero VEV here, is then forced[11] to be parallel to $\langle\phi_r\rangle$, that is, $\langle A_{ry}\rangle = v_A \lambda_6$, by the minimization condition of the Hosotani potential[7]. Note that both $\langle\phi_r\rangle$ and $\langle A_{ry}\rangle$ are of the order R^{-1}.

As a result, for the down sector, we have the following mass matrix:

$$\left(\bar{d}_{rR} \quad \bar{B}_{lR} \right) \begin{pmatrix} \hat{f}_\Sigma v_\sigma & \hat{f}_r v_r + i g_r v_A \hat{\mathbf{1}} \\ \hat{f}_l v_l & \hat{M}_D \sim \hat{f}_\Sigma \langle\chi\rangle \end{pmatrix} \begin{pmatrix} d_{lL} \\ B_{rL} \end{pmatrix}, \tag{5}$$

where $\hat{f}_l$, $\hat{f}_r$, $\hat{f}_\Sigma$ denote the Yukawa coupling matrices of ϕ_l, ϕ_r, Σ, respectively. Note that the Hosotani term is generation independent and proportional to the unit matrix $\hat{\mathbf{1}}$. Generically, this fermion mass matrix will give complex phases and lead to at least Kobayashi-Maskawa(KM) type of CP violation. Note that the up quark mass matrix is purely from $\hat{f}_\Sigma v_\sigma$ which is real and does not contribute to the CP violating phase.

The down quark mass matrix above have some similarity with that in a model proposed to solve the strong CP problem [12]. Unfortunately the current model as it is does not provide a solution to strong CP problem. However, an extension of the model with flavor symmetry may be able to achieve this goal.

Note that we assumed that $\langle A_l\rangle$ vanishes because we don't want to use Hosotani term to break $SU(2)_l$ gauge symmetry. We want to break the $SU(2)_l$ group at a scale lower than the compactification scale through the Higgs mechanism as in the Standard Model.

NEUTRINO MASS

In the lepton sector, $\langle\phi_r\rangle$ will give large masses to two of the three $SU(3)_l$ triplets and leave one triplet per generation in L at the usual $SU(2)_l$ scale which serve as the usual light leptons. The v_σ will give rise to Dirac masses for the lepton. The model as it is still have massless neutrinos.

Let's consider the leptonic sector in more detail. We fix the convention such that $L \to U_l^+ L U_r, \bar{L} \to U_r^+ \bar{L} U_l$, and $\Phi_r \to U_r^+ \Phi_r U_r$.

The mass term of the leptons are coming from three types of terms: $Tr(\bar{L}L\Phi_r), Tr(\bar{L}\Phi_l L)$ and $LL\Sigma$ [13].

If we write

$$L \sim \begin{pmatrix} N_L^0 & E_{2L}^+ & E_{2R}^+ \\ E_{1L}^- & N_{1L} & N_{1R} \\ E_{1R}^- & N_{2R} & N_{2L} \end{pmatrix} \tag{6}$$

and

$$\Sigma \sim \begin{pmatrix} S_{11}^0 & S_{12}^+ & 0 \\ S_{21}^- & S_{22}^0 & 0 \\ 0 & 0 & \chi \end{pmatrix}, \tag{7}$$

we get the following interacting terms from $LL\Sigma$:

$$\begin{aligned} LL\Sigma \supset \quad & N_{1L}N_L^0\chi - E_{1L}^- E_{2L}^+\chi \\ + \; & N_{1L}N_{2L}S_{11}^0 - N_{1R}N_{2R}S_{11}^0 \\ + \; & N_L^0 N_{2L} S_{22}^0 - E_{1R}^- E_{2R}^+ S_{22}^0 \\ + \; & N_{2R}E_{2R}^+ S_{21}^- - N_{2L}E_{2L^+}S_{21}^- \\ + \; & N_{1R}E_{1R}^- S_{12}^+ - N_{2L}E_{1L}^- S_{12}^+. \end{aligned} \tag{8}$$

Combining with terms from $Tr(\bar{L}L\Phi_r), Tr(\bar{L}\Phi_l L)$, we get the following charged lepton mass matrix, in the basis of $(\bar{E_{1R}^-}, \bar{E_{2L}^+}, \bar{E_{1L}^-}, \bar{E_{2R}^+})$,

$$M_{charged} \sim \begin{pmatrix} 0 & 0 & <\phi_{l6}> & <S_{22}^0> \\ 0 & 0 & <\chi> & <\hat{\phi}_{r6}> \\ <\phi_{l6}> & <\chi> & 0 & 0 \\ <S_{22}^0> & <\hat{\phi}_{r6}> & 0 & 0 \end{pmatrix} \tag{9}$$

while the neutral lepton mass matrix, in the basis of $(N_L^0, \bar{N_{1R}}, \bar{N_{2R}}, N_{2L}, N_{1L})$, is of the form,

$$M_{neutral} \sim \begin{pmatrix} 0 & 0 & 0 & <S_{22}^0> & <\chi> \\ 0 & 0 & <S_{11}^0> & <\phi_{l6}> & <\hat{\phi}_{r6}> \\ 0 & <S_{11}^0> & 0 & <\hat{\phi}_{r6}> & <\phi_{l6}> \\ <S_{22}^0> & <\phi_{l6}> & <\hat{\phi}_{r6}> & 0 & <S_{11}^0> \\ <\chi> & <\hat{\phi}_{r6}> & <\phi_{l6}> & <S_{11}^0> & 0 \end{pmatrix} \tag{10}$$

where $<\hat{\phi}_{r6}>$ is a linear combination of $<\phi_{r6}>$ and $i<A_{ry}>$ and give rise to CP violation in the lepton sector. The Yukawa couplings in these matrices are ignored since they are meant to indicate only the scale of the respective terms. Note that $<S_{11}^0>, <S_{22}^0>$ and $<\phi_{l6}>$ are $SU(2)_l$ breaking scale and should be small compared with $<\chi>$ or $<\phi_{r6}>$ which are $SU(2)_r$ breaking scale or higher. To first approximation, we can set $SU(2)_l$ breaking scale to zero and we find that there are two zero eigenvalues in the charged mass matrix and only one zero eigenvalue in the neutral one. That means that we have one vectorial pair of massless charged leptons and one massless neutral chiral lepton. Turning on $<S_{11}^0>, <S_{22}^0>$ and $<\phi_{l6}>$ will make the determinants nonzero and we naturally have a see-saw structure in the neutral lepton mass matrix. That means that we have a pair of light charged leptons and a super-light neutral lepton (we identify it as the neutrino per generation) due to see-saw mechanism.

DISCUSSIONS

The problem of using Hosotani mechanism to break gauge symmetry and generate masses is that the natural value of the generated mass scale is of order of compactification scale. This is because the Hosotani mechanism makes use of the phase factor of the Wilson line integral $\exp(ig_r \int A_{ry}dy)$ which in term gives $g_r\langle A_r\rangle \sim O(\frac{1}{R})$, where g_r is the gauge coupling constant of $SU(2)_r$ and R is the radius of the extra dimension.

To decouple the Hosotani breaking of CP symmetry with the lower energy $SU(2)_l$ breaking, we have constructed a realistic model which can generate CP violations even though all the Yukawa couplings and all the Higgs VEVs are real. Furthermore, we can have see-saw type neutral lepton mass matrix naturally.

One important feature of our model is that we use Hosotani term $g_r\langle A_r\rangle$ to break $SU(2)_r$ gauge symmetry and to** generate CP violation at the same time! The breaking of gauge symmetry and that of CP are related to each other and they are both originated from the existence of extra dimensions. In addition, by assigning proper gauge and Z_2 quantum numbers for each fields in the $4+1$ dimensional theory, we have a natural way to provide the desired chiral state in the $3+1$ dimensions.

So, we manage to account for a $3+1$ dimensional theory which is effectively KM-like in nature starting from a higher dimensional CP conserving theory using gauge as well as CP breaking Hosotani mechanism. This may provide some insight to the origin of CP violation. A few questions immediately arise. Is strong CP problem of the KM model resolved in this model? The question is unfortunately no. However, the mechanism used here seems to provide enough flexibility that one suspects that there may be similar models properly extended that can solve the strong CP problem. Another question is the new physics that may arise related to this mechanism. The new physics scale R^{-1} can in principle be very high

such as 10^{10-11} GeV if one uses the see-saw mechanism to explain the small observed neutrino masses. In that case, the CP violating scale, the compactification scale and the neutrino see-saw scale are all tie together which is interesting. On the other hand, if one first ignores the lepton sector, since it is more remote to the observed CP phenomena in the quark sector, or if one allows fine-tuning to make the neutrino masses small, then the scale R^{-1} can be as small as the experimental limit on the light vectorial quark mass. Since such vectorial quark can in principle violate the unitarity of the quark mixing matrix, one can derive a limit of around a few TeV based on the experimental limit on the unitarity of KM mixing matrix[12]. If that is the case, one expects** to have a wealth of new CP violating phenomena in the next generation collider experiments at LHC or NLC. This will be investigated in detail in the future. Another interesting problem to be looked into more carefully in the future is the new CP violating phenomena predicted in this model.

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[9] One can in principle insert a phases for each flavor of fermion, $\Psi(x_\mu, y)_i = \eta_i P_{\mathcal{G}} \gamma_5 \Psi(x_\mu, -y)_i$, where ($|\eta|^2 = 1$), but we do not find it necessary in current model.

[10] We assume that the Z_2 transformation property between Q_l and $\bar{Q}_r$ has a relative phase such that $\eta_R^* \eta_L = -1$.

[11] The minimization of the scalar potential coming from the scalar kinetic term : $g^2[\langle A_y\rangle, \langle\phi_{r6}\rangle]^2 \subset |D_y\phi_r|^2$ means that $\langle A_y\rangle$ and $\langle\phi_{r6}\rangle$ must commute with each other. Since all diagonal component $\alpha\langle A_{r3y}\rangle + \beta\langle A_{r8y}\rangle$ are forbidden by orbifold condition, only $\langle A_{6y}\rangle$ can non-vanishing VEV, see [7].

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[13] Write explicitly, this is $\epsilon_{lmn}\epsilon^{ijk} L_i^l L_j^m \Sigma_k^n$.